CATS5 - Titles and abstracts

School

- **Matthew Morrow:** *Rigidity properties of algebraic K-theory and topological cyclic homology.*

  This mini-course will be an introduction to certain rigidity" properties of K-theory and topological cyclic homology, both classical and modern. For example, Suslin rigidity asserts that K-theory (with finite coefficients away from the characteristic) is invariant under extensions of separably closed fields, while Gabber rigidity asserts that K-theory (again with finite coefficients away from the characteristic) is invariant when modding out a ring by a Henselian ideal. Replacing K-theory by the difference between K-theory and topological cyclic homology, analogous statements hold even at the characteristic. The goal will be to introduce the audience to these types of results and the methods used to establish them, such as reduction via finiteness properties of K-theory and TC to the case of smooth algebras and the explicit calculations needed in that case. Much of the course will be based on recent joint work with Dustin Clausen and Akhil Mathew.

- **Mauro Porta:** *Derived methods in analytic geometry.*

  This will be mostly an introductory course to derived geometry with a special attention to its analytic variants and applications. It will consist of three lectures.

  During the first lecture I will mostly focus on giving examples of situations where techniques from derived geometry can be successfully applied. The main examples I will develop here are the comparison between the Hochschild homology and the de Rham cohomology in the singular situation (HKR theorem) and the study of certain classical moduli problems that become simpler after the passage to the derived world.

  During the second lecture I will provide more foundational background on derived geometry. While the first lecture will only require some standard knowledge in homological algebra and algebraic geometry, in this lecture I will draw upon the machinery of $\infty-$categories. I will present several fundamental properties of derived schemes that make the theory workable in practice.

  During the third lecture I will turn my attention to the analytic world, which means both complex analytic and non-archimedean analytic. I will give some example of moduli problem that is naturally representable analytically but not algebraically and whose derived structure plays a role in current research. Next I will sketch how to extend the formalism and the properties discussed during the second lecture in order to deal with the analytic situation.

- **Hiro-Lee Tanaka:** *Morse theory through a stack of broken lines.*

  In this short course, we'll construct a stack that gives a way of encoding all associative algebras (in the following sense: giving a factorizable sheaf on this stack is the same thing as giving a not-necessarily-unital Aoo-algebra). This stack has two equivalent interpretations–it's the stack of possibly broken gradient trajectories on a point, or the stack parametrizing the domains of broken trajectories. Moreover, by taking compactly supported sections of an Aoo-algebra in a factorizable way, we expect to recover the Koszul dual of the algebra. This formalism allows us to construct Morse theory over any coefficient ring, and is one baby step toward constructing Lagrangian Floer theory over arbitrary coefficients (e.g., over stable homotopy theory). All this is joint with Jacob Lurie.

Conference

- **Mohammed Abouzaid:** *HMS for local models of toric degenerations.*

  The work of Gross and Seibert constructing mirror pairs associated to toric degenerations of Calabi-Yau manifolds specifies a class of local models from which general examples are constructed via an elaborate "wall-crossing" procedure. I will explain a proof of homological mirror symmetry for these local models, which relies on the use of immersed Lagrangians, and on explaining the mirror phenomenon as a computation of a (component) of a moduli space of objects in the Fukaya category.

- **Benjamin Antieau:** *Cartier modules and cyclotomic spectra.*

  I will describe a t-structure on cyclotomic spectra which builds a bridge to the theory of Cartier modules and p-divisible groups. As an example, I will describe how computations of Hesselholt allow one to identify the lowest non-trivial cyclotomic homotopy group of (the topological Hochschild homology of) a K3 surface X over a perfect field of characteristic p as the formal Brauer group of X. This is joint work with Thomas Nikolaus.

- **Damien Calaque:** *Shifted symplectic reduction of derived critical loci.*

  In this talk we will introduce the notions of shifted symplectic groupoid and shifted symplectic reduction. We will provide examples, focusing in particular on describing equivariant derived critical loci by means of shifted symplectic reduction. If time permits, we will also explain how one can construct shifted symplectic groupoids from the AKSZ-PTVV construction. This is based on joint works with Mathieu Anel (equivariant derived critical loci) and Pavel Safronov (shifted symplectic groupoids).

- **Frédéric Déglise:** *Syntomic modules.*

  It now well established that Voevodsky's category of motives satisfies the universal property, at the homotopical level, that Grothendieck had envisioned. Well-behaved homotopical realizations have been constructed, among which the p-adic tale realization which is the most general. On the other hand, Fontaine's p-adic Hodge theory aims at describing p-adic Galois representations of "geometric origin", meaning coming from an (abelian) motive. This relies on elaborated relations between various p-adic cohomologies, which was ultimately extended by Alexander Beilinson. The problematic of this talk, addressed in a collaboration with Wieslawa Niziol, is to link these two domains. Relying on Beilison's ideas, we build a homotopical version of p-adic Hodge theory which allows us to get a "syntomic ring spectrum", a kind of derived version of Fontaine's period rings. We then show that modules over this ring spectrum, in the sense of motivic homotopy theory, can be identified with Fontaine's p-adic representations (explicitly: the semi-stable ones). I will then describe how it allows us to build a relative version of Fontaine's theory.

- **Alexander Efimov:** *Localizing invariants for large categories.*

  We will explain how to extend (in a natural way) a localizing invariant F of small spectral categories to the class of dualizable cocomplete categories, so that for compactly generated categories we get the the value of F on compact objects. The construction uses Calkin categories.

  When a localizing invariant commutes with filtered colimits, we will sketch the (highly non-trivial) computation of its value on the category of sheaves of modules over a presheaf of E_1-rings on a locally compact Hausdorff space.

- **Benjamin Hennion:** *Gelfand–Fuks cohomology of algebraic varieties.*

  (joint work with M. Kapranov) Given a smooth affine algebraic variety over the complex numbers, we compute the Chevalley-Eilenberg cohomology of its Lie algebra of global vector fields, we show it is a topological invariant of the underlying complex manifold and is finite dimensional in every degree. The proof uses methods from factorization homology in both topological and algebraic flavors. In this talk, we will first explain the case of smooth real manifolds as studied in the 70's (Gelfand, Fuks, Bott–Segal, Haefliger, Guillemin, ...). We will show how to transpose those methods to complex algebraic varieties.

- **Marc Hoyois:** *Moduli stacks of varieties and algebraic cobordism.*

  A "moduli stack of varieties" M is a functor associating to every scheme X some groupoid M(X) of schemes over X. An object of $M(A^1)$ is thus a scheme over the affine line A^1, which we can interpret as a cobordism between its fibres over 0 and 1. The A^1-homotopy type of such a moduli stack is thus a naive algebro-geometric analog of cobordism spaces in algebraic topology, which are closely related to Thom spectra by a famous theorem of Thom. I will present a version of Thoms theorem in algebraic geometry, which states that the A^1-homotopy type of the moduli stack of proper 0-dimensional local complete intersections is the motivic Thom spectrum MGL defined by Voevodsky. This is joint work with Elden Elmanto, Adeel Khan, Vladimir Sosnilo, and Maria Yakerson.

- **Dominic Joyce:** *Vertex algebra structures on the homology of moduli stacks.*

  Let A be a nice abelian or triangulated category, such as $coh(X)$ or $D^b_{coh}(X)$ for X a smooth projective variety over C, let M be the (higher) stack of objects in A, and let $H_*(M)$ be the homology of M over a commutative ring R. After choosing a small amount of extra data on M, for which there are natural choices in examples, I explain how to define the structure of a graded vertex algebra over R on $H_*(M)$. Vertex algebras are complicated, infinite-dimensional algebraic structures coming from Conformal Field Theory in Physics. There must be a physical explanation for these vertex algebras, but I dont know it. The construction is most natural when A is a Calabi-Yau even category, so that M is even shifted symplectic in the sense of Pantev-Toen-Vaquie-Vezzosi. For non-Calabi-Yau categories, morally we produce examples by replacing M by $T^*M[2n]$, which is even shifted symplectic, with $H_*(M) = H_*(T^*M[2n])$. If $A = D^b mod - CQ$ for Q a quiver then $H_*(M)$ is the lattice vertex algebra associated to the symmetrized generalized Cartan matrix of the quiver. The construction seems to be related to a lot of interesting mathematics: it may explain Grojnowski-Nakajima's representations of Heisenberg algebras on homology of Hilbert schemes of surfaces; there are connections to Ringel-Hall algebras and Cohomological Hall algebras; I discovered the vertex algebra structure whilst studying wall-crossing formulae for Donaldson-Thomas type invariants of Calabi-Yau 4-folds. The construction can also be generalized in lots of different directions.

- **Ludmil Katzarkov:** *Categorical Spectral covers and curve complexes.*

  In this talk we will introduce some categorical analogues of classical geometric notions. Applications will be discussed.

- **Akhil Mathew:** *Deformation theory and partition Lie algebras.*

  Abstract: A theorem of Lurie and Pridham states that over a field of characteristic zero, derived "formal moduli problems" (i.e., deformation functors defined on derived Artinian commutative rings), correspond precisely to differential graded Lie algebras. This formalizes a well-known philosophy in deformation theory, and arises from Koszul duality between Lie algebras and commutative algebras. I will report on joint work (in progress) with Lukas Brantner, which studies the analogous situation for arbitrary fields. The main result is that formal moduli problems are equivalent to a category of "partition Lie algebras"; these are algebraic structures (which agree with DG Lie algebras in characteristic zero) which arise from a monad built from the partition complex.

- **Aaron Mazel-Gee:** *The secondary cyclotomic trace.*

  The Dennis trace is a natural map K –¿ HH from algebraic K-theory (K) to Hochschild homology (HH), an algebro-geometric analog of the Chern character. There is a refinement of HH called "topological cyclic homology" (TC), and the Dennis trace refines to give the cyclotomic trace K –¿ TC. Introduced over 25 years ago, the cyclotomic trace is a strikingly effective means for computing higher algebraic K-groups (in fact it remains our only means of doing so), but it has so far resisted a geometric interpretation. I will describe recent joint work with David Ayala and Nick Rozenblyum, in which we give a precise interpretation of TC and the cyclotomic trace at the level of derived algebraic geometry. I will also describe joint work in progress with Reuben Stern, in which we produce a 2-dimensional version of this story: the secondary cyclotomic trace map, which runs from secondary algebraic K-theory (which is built from sheaves of categories instead of sheaves of vector spaces) to a 2-dimensional analog of TC.

- **Takuro Mochizuki:** *Stokes shells and Fourier transform.*

  Recently, there has been much concern about the irregular singularities of meromorphic flat connections, or more generally holonomic D-modules. The irregular singularities are essentially classified by their topological counterpart called Stokes structure. They are attractive because the Stokes structures of some D-modules contain much interesting information in various fields of mathematics including algebraic geometry and mathematical physics.

  In this talk, we shall discuss the Stokes structure of the holonomic D-modules obtained as the Fourier transform of holonomic D-modules on an affine line. One of the main issues is to introduce an appropriate formulation of Stokes structures which is suitable to the study of the Fourier transform.

- **Johannes Nicaise:** *Specialization of (stable) rationality in families with mild singularities.*

  I will present joint work with Evgeny Shinder, where we use Denef and Loeser's motivic nearby fiber and a theorem by Larsen and Lunts to prove that stable rationality specializes in families with mild singularities. I will also discuss an improvement of our results by Kontsevich and Tschinkel, who defined a birational version of the motivic nearby fiber to prove specialization of rationality.

- **Pavel Safronov:** *Noncommutative Poisson geometry and the Kashiwar-aVergne problem.*

In this talk I will explain an interpretation of the KashiwaraVergne problem (a property of the BakerCampbellHausdorff series) in terms of noncommutative geometry. Namely, one can reformulate it as a formality statement for the CalabiYau algebra of cochains on a Riemann surface equipped with a trivialization of the Euler class. In the talk I will also describe noncommutative versions of familiar concepts such as shifted Poisson structures and their unimodular versions. This is a report on work in progress with Florian Naef.

- **Sarah Scherotzke:** *The Chern character and categorification.*

  The Chern character is a central construction which appears in topology, representation theory and algebraic geometry. In algebraic topology it is for instance used to probe algebraic K-theory which is notoriously hard to compute, in representation theory it takes the form of classical character theory. Recently, Toen and Vezzosi suggested a construction, using derived algebraic geometry, which allows to unify previous Chern characters. We will categorify the Chern character. In the categorified picture algebraic K-theory is replaced by the category of non-commutative motives. The categorified Chern character has many interesting applications such as proving that the DeRham realisation functor is of non-commutative origin.

- **Vivek Shende:** *User's guide to the wrapped Fukaya category.*

  I will describe some tools for manipulating wrapped Fukaya categories without getting one's hands dirty with holomorphic disk calculations all the time.

- **Carlos Simpson:** *Stability conditions and spectral networks for A_5 tensor A_2.*

  We present ongoing joint work with Haiden, Katzarkov and Pantev on the construction of a stability condition for the Fukaya category on the plane, with six marked points at the vertices of a regular hexagon, with coefficients in the A_2 category. The semistable objects are represented by spectral networks that can be described explicitly in this case.

- **Tony Yu Yue:** *Enumeration of non-archimedean curves in higher dimensional log Calabi-Yau varieties.*

  I will discuss the enumeration of non-archimedean curves in higher dimensional affine log Calabi-Yau varieties containing an open algebraic torus, part of my joint work with S. Keel. This generalizes the previously studied two-dimensional case, and includes cluster varieties arising from representation theory. Many new ideas are developed in order to go beyond the two-dimensional case. In my talk, I will explain various properties of the moduli spaces which lead to the enumeration. Moreover, I will introduce a new notion of skeletal curves, curves whose skeleton lies in the essential skeleton of the ambient log Calabi-Yau variety. Such curves play a special role in the theory.